

English 'If-Then' Statements in Propositional Logic

Using arbitrary letters to represent two sentences, *if-then* statements in English can be represented very basically as “If p , then q .” With this as a guideline, the same *if-then* statement can be represented in propositional logic as $p \rightarrow q$. This brings us to the question: Can English *if-then* statements accurately be recreated using propositional logic alone? I show that propositional logic can in fact paraphrase some English *if-then* statements. However, a certain pragmatic use of *if-then* statements in English will show us that they cannot successfully be recreated in propositional logic while keeping their coherence.

Shown below is the truth table for the proposition $p \rightarrow q$, where 1 means *true* and 0 means *false* (relevant columns are in italics).

p	q	$p \rightarrow q$
0	0	0 1 0
0	1	0 1 1
1	0	1 0 0
1	1	1 1 1

From this table, it can be seen that the proposition $p \rightarrow q$ is only false when its antecedent (p) is true, and its consequent (q) is false. This can be illustrated quite easily in English with a few example sentences:

- 1 If you study well, then you will pass the exam.
- 2 If you eat, then you will feel full.

Example (1) will only be false when a person studies well, and still fails his/her exam. In other words, “If you study well” has a value of 1, and “you pass the exam” has a value of 0. In this case, propositional logic is successful in representing an English *if-then* statement.

In propositional logic, the contrapositive ($\neg q \rightarrow \neg p$) will maintain the truth values of the original proposition ($p \rightarrow q$), just as it works in English (sometimes; I will show why it doesn't always work later). To represent the contrapositive in propositional logic, I will use the symbol \neg meaning “not” in the following table:

p	q	$p \rightarrow q$	$\neg q \rightarrow \neg p$
0	0	0 1 0	1 0 1 1 0
0	1	0 1 1	0 1 1 1 0
1	0	1 0 0	1 0 0 0 1
1	1	1 1 1	0 1 1 0 1

As can be seen, the contrapositive maintains the same truth values as its original proposition. In English, this can be shown with similar examples:

- 3 If you didn't pass the exam, then you didn't study well.

4 If you don't feel full, then you didn't eat.

From (3) and (4), it holds that the contrapositive of an *if-then* statement can also be successfully represented using propositional logic, since its truth values do not change when we manipulate the symbols to follow the rules of a contrapositive.

However, it is not always the case that the contrapositive of an English sentence will maintain its original truth values. Certain *if-then* statements in English only make sense to us when they're used in raw form pragmatically, and become nonsensical when we look at their contrapositives. To prove this, I address the following example:

5 If you liked that movie, then I have another one we can watch.

Sentence (5) is a perfectly acceptable English *if-then* sentence, except it's not one that adheres so nicely to propositional logic. When its contrapositive is taken, it quickly becomes a sentence which would baffle any native speaker of English, as in (6):

6 If I don't have another one we can watch, then you didn't like that movie.

From this we can see that propositional logic is not a sound method for representing *if-then* statements in English. While it can handle straightforward sentences very well, it's when we try to represent pragmatic sentences which rely on context, that its shortcomings are spotted.