

Final Squib

Simple sentences of English can be analyzed using predicate formulas, and depending on the structure of the sentence (e.g. number of adjectives, whether a relative clause is included, etc.), the structure of its predicate counterpart will change accordingly. Specifically, the addition of a relative clause, for example, will change the way nouns in the sentence are represented using arbitrary variables like x and y . The order of adjectives in a sentence also has a bearing on the final meaning of the sentence.

1a John is old.

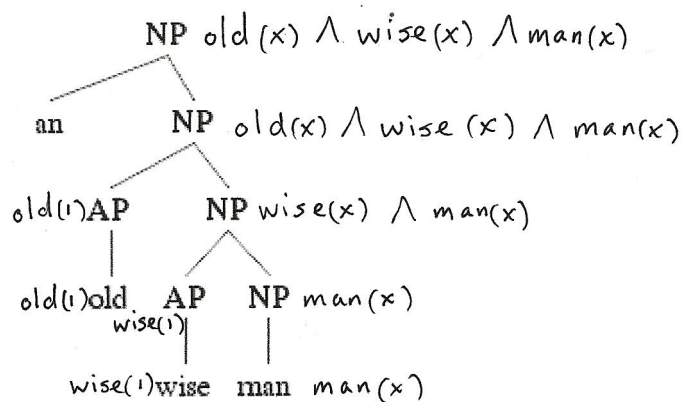
1b $old(John)$

In a sentence such as Example 1a, the most basic of the predicate templates, we've demonstrated that it can be evaluated and represented (in this case) as in Example 1b. The adjective, *old*, accepts the subject of the sentence, *John*, as an argument to be modified.

2 An old man.

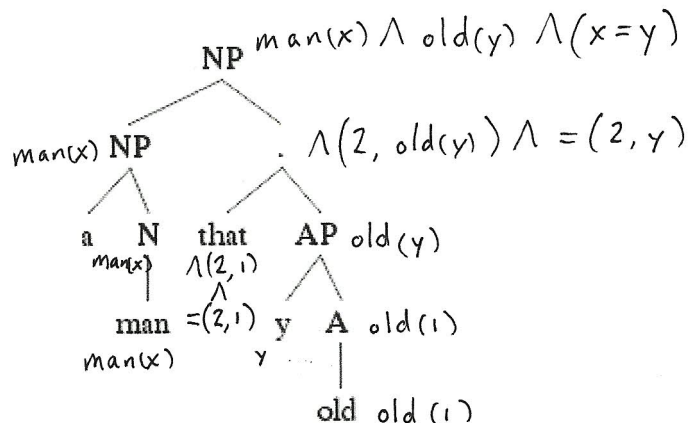
Example 2, however, is slightly different from Example 1 in that it contains an instance of an adjective directly describing some arbitrary man, 'man'. In this case, we need to add a variable, x , in order to link two predicates that modify the same subject, and we need to make use of the logical operator, \wedge . This evaluates to $old(x) \wedge man(x)$.

3 An old, wise man.



Example 3 can be described as a slight extension of the case in Example 2. In Example 2, we had an instance of two predicates that modified the same subject (which we assigned to a variable, x). In this new case, there are two adjectives that describe some arbitrary man. We can extend our predicate formula with another logical operator (\wedge) and we're left with $old(x) \wedge wise(x) \wedge man(x)$. This formula results from the addition of a second AP with the adjective 'wise' in the D-structure of our sentence. Again, all three predicates in our formula modify our subject, x .

4 A man that is old.



In Example 4, we see the addition of a relative clause by the use of the word ‘that’. What this means for our predicate evaluation is that we need to add a second variable while keeping our original x . The reason for this is illustrated by Example 4’s D-structure; the word ‘that’, at its node in the D-structure, functions as a link for the variable y in $old(y)$ and the variable x in $man(x)$. The second variable, y , will refer to the same subject in the sentence, which signals that there is a relative clause. The noun phrase of this sentence remains $man(x)$. What changes, however, is the second half of the sentence containing the adjective phrase. We can use another arbitrary variable, y , and use it as the argument for the adjective, old . This leaves us with $man(x) \wedge old(y)$. This, however, isn’t complete; we need to add something to the formula that lets us know that the two variables, x and y , refer to the same subject. This sentence evaluates to $man(x) \wedge old(y) \wedge (x = y)$.

These fundamental examples of predicate formulas for sentences with adjectives don’t solve a problem that arises with certain adjectives – ‘old’ being one of these cases. An adjective such as ‘old’ will convey a meaning that varies with the noun it describes. Crucially, *an old pastry* is not nearly as old as *an old man*. In our current system, predicates for adjectives such as ‘old’ are unable to distinguish, in this example, between degrees of ‘oldness’. Regardless, the same adjective, ‘old’, is used without any issue of misunderstanding to a native speaker of English. In our predicate evaluations, we can try to mitigate this problem by adding an argument in the adjective’s parentheses. Now, there will be two arguments, whereas previously there was only one (the variable, x).

To recycle and modify Example 2, we can use the new predicate formula $old(x, man) \wedge man(x)$. What changed here is the addition of the second argument, ‘man’, in the predicate $old()$. Now, the reader is able to understand that x is old with respect to ‘man’, and not old with respect to anything else, like ‘pastry’. ‘Old’, being a scalar adjective, introduces a problem of representation of meaning. Without this modified system, the meaning of ‘old’ and similar adjectives cannot be handled well enough for our purposes. Before this modification, the degree of ‘oldness’ (for this example) in relation to ‘man’ wouldn’t have been specified. In the case of ‘pastry’, we can simply switch the predicates and arguments while maintaining the structure: $old(x, pastry) \wedge pastry(x)$ which is equivalent to ‘an old pastry’.

Using this modified system, we can address a case of two sentences (Examples 5a and 6a below) that one might assume have the same meaning, but actually refer to two different things.

5a A small expensive house.

6a An expensive small house.

Having two of the same adjectives modifying the same noun is what might signal that these two sentences are identical in meaning. However, we'll see that the order in which the two adjectives (*expensive* and *small*) modify the noun (*house*) has a strong influence on the final meaning of the two sentences. Using the modified predicate system, Examples 5a and 6a can be represented as in 5b and 6b below, respectively:

5b $small(x, expensive\ house) \wedge expensive(x, house) \wedge house(x)$

6b $expensive(x, small\ house) \wedge small(x, house) \wedge house(x)$

Based on our previous example with 'old', we can derive the meaning for these formulas in the same way; in 5b, for example, $small(x, expensive\ house)$ tells us that 'x is small with respect to the usual size of expensive houses' (hence *expensive house* as the second argument in the predicate, used for reference to show comparison). Then, $expensive(x, house)$ tells us that 'x is expensive with respect to the usual cost of houses, in general'. Finally, from $house(x)$, we can understand that x is indeed a house, and the prior predicates in the formula now clearly contribute to the intended meaning for 5a: 'an expensive house that is small in terms of the average expensive house'.

We can also look at a slightly different case than Examples 5a and 6a. Examples 7 and 8 below seem not to follow the same pattern as Examples 5a and 6a. That is to say, their meanings cannot be evaluated to mean two different things; both, because of the addition of 'and' in each, are identical in meaning (intuitively, to a native speaker of English).

7 A small and expensive house.

8 An expensive and small house.

Since the adjectives in Examples 7 and 8 are separated by 'and', their D-structures are similar to the D-structure of Example 3. In this case, however, 'old' and 'wise' are replaced with 'small' and 'expensive', and 'man' is replaced with 'house'. Specifically, each of the two adjectives describe the noun separately and without influence from the other adjective. In other words, the *house* is *small* with respect to the usual size of houses, while also being *expensive* with respect to the usual cost of houses.

To take the case with the examples involving 'and' further, we can compare Examples 7 and 8 with Examples 5a and 6a. The meaning derived from Examples 7 and 8 (their formulas in 7b and 8b below) can possibly be the same as the meaning of Example 6a, but not 5a.

7b $small(x) \wedge expensive(x) \wedge house(x)$

8b $expensive(x) \wedge small(x) \wedge house(x)$

Crucially, the order of the first two predicates, $small(x)$ and $expensive(x)$, in both formulas doesn't matter. Since they're separated by \wedge , the order in which they modify x has no bearing on the meaning of the sentence.

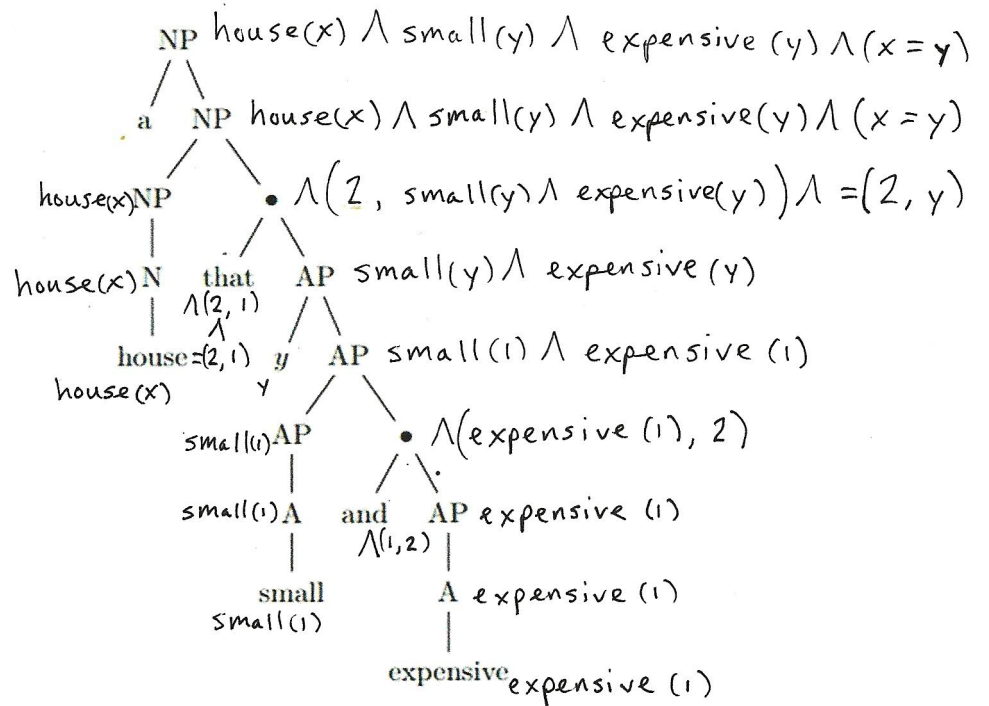
In Example 6a, we have a house that is small ('small house') being modified by the adjective 'expensive', meaning 'a small house that is expensive in terms of the usual cost of small houses'. According to native speaker intuitions, this necessarily means that the house in question is both small and expensive, as in the meaning of Examples 7 and 8.

Now, we can analyze similar examples that contain relative clauses, to see how their meanings line up with our analysis:

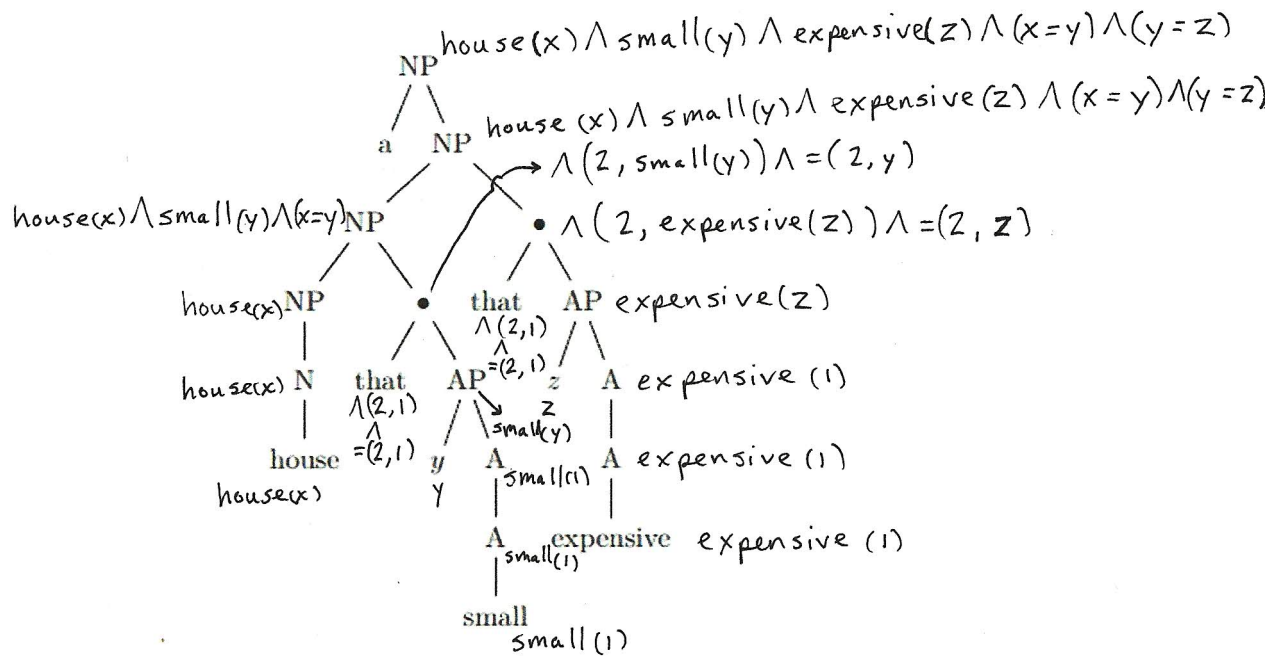
9 A house that is small and expensive.

10a A house that is small that is expensive.

11 A house that is expensive that is small.



Example 9's D-structure (above), while it includes a relative clause, happens to have the same meaning as that of Examples 7 and 8. Since there is only one relative clause here, and its adjectives are separated by 'and' like in Examples 7 and 8, its meaning is the same.



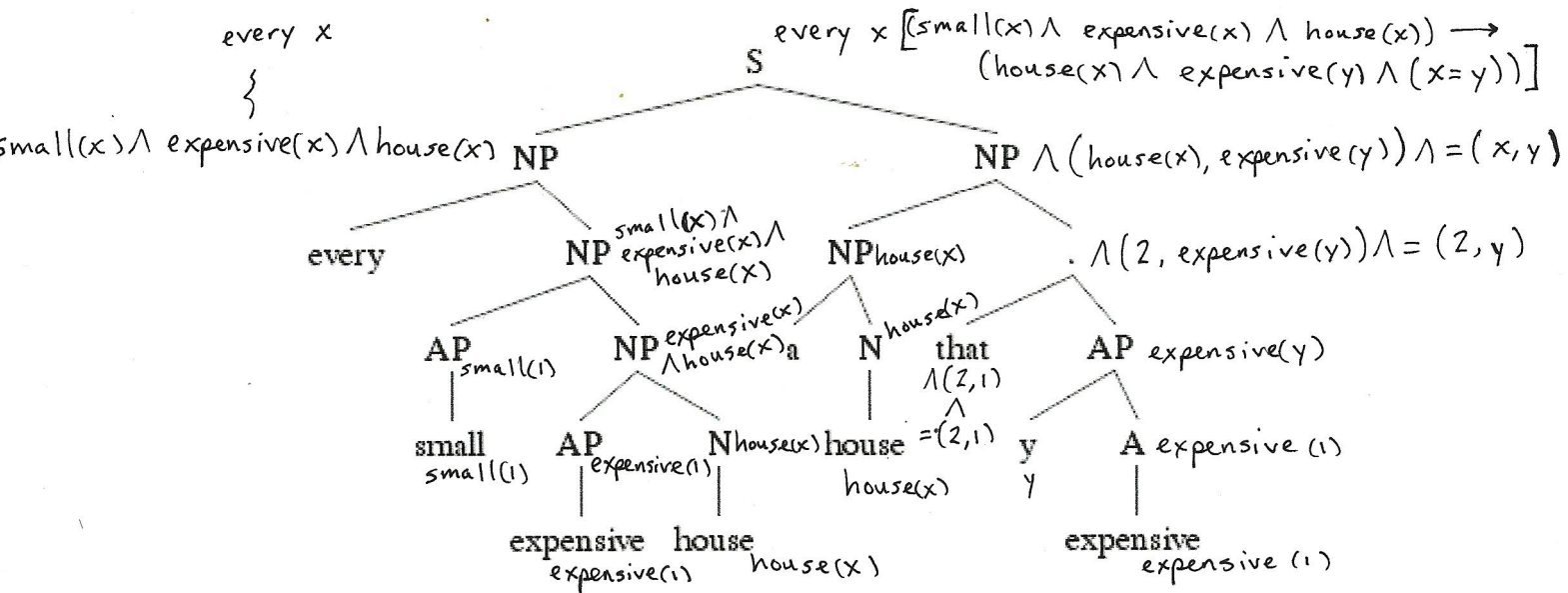
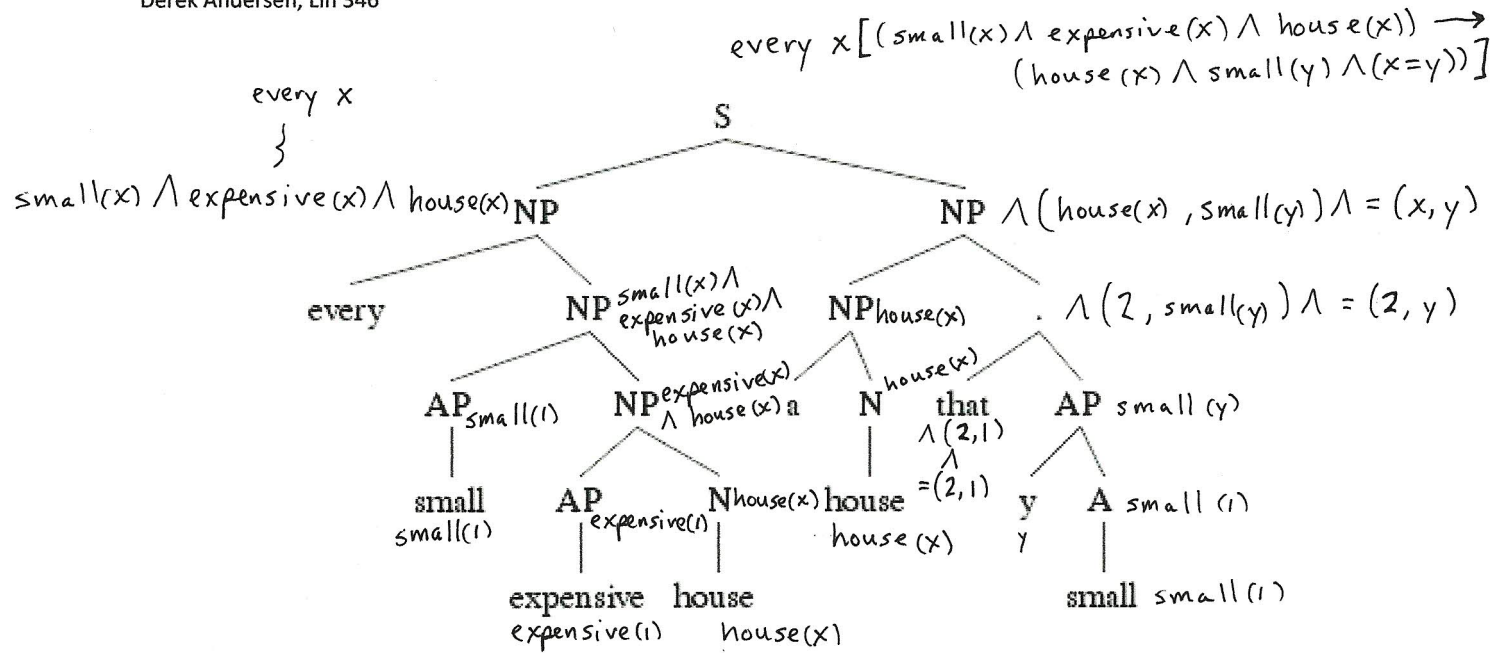
Above is the D-structure for Examples 10a and 11 (but with 'small' and 'expensive' switched for Example 11). The meaning for Example 10a follows the meaning for Example 6a, while Example 11 follows the meaning for Example 5a. The order of the relative clauses in the sentence and D-structure determines the final meaning for the sentence. Semantically, 'a house that is small' from Example 10a is modified further by 'that is expensive', making the small house expensive, or 'a small house that is expensive in terms of the usual cost of small houses' (as in Example 6a's meaning). The predicate formula for Example 10a (below in 10b) is further evidence for its relation to Example 6a's meaning:

10b $house(x) \wedge small(y, house) \wedge expensive(z, small\ house) \wedge (x=y) \wedge (y=z)$

Based on these analyses, we can look at two more sentences to determine whether they are true in any arbitrary context:

- 12a Every small expensive house is a house that is small.
- 13a Every small expensive house is a house that is expensive.

The trees for Examples 12a and 13a are below, respectively:



Intuitively, Example 12a is false in some context while Example 13a is true in any context. The universal quantifier 'every' lets us analyze every arbitrary *small expensive house* in both examples. However, each example varies by its second half; a different relative clause modifying the 'small expensive house'. To be an *expensive house* before being modified by *small*, the house in question, intuitively, must not be small. So, the statement 'every small expensive house is a house that is small' is necessarily false in any arbitrary context. On the other hand, an *expensive house*, regardless of its smallness with respect to the usual size of an expensive house, must necessarily be expensive. This can be reinforced with Example 12a and 13a's predicate formulas below, in Examples 12b and 13b.

12b $every\ x\ [(small(x) \wedge expensive(x) \wedge house(x)) \rightarrow (house(x) \wedge small(y) \wedge (x=y))]$

13b $every\ x\ [(small(x) \wedge expensive(x) \wedge house(x)) \rightarrow (house(x) \wedge expensive(y) \wedge (x=y))]$

After developing a system to mitigate the problem of the “degree” of adjectives in sentences (by using an additional argument in the predicate formula) we are able to better express the meaning of, for example, ‘an old man’. The order in which adjectives modify nouns in a sentence’s D-structure has an influence on the final meaning of the sentence. In the case of two sentences whose multiple adjectives modify the same noun, switching the adjectives when the word ‘and’ is absent can affect their meanings. The addition of a relative clause also affects the outcome of a sentence’s meaning and D-structure.