Large-scope English 'If-Then' Statements in Propositional Logic

To represent three distinct statements of English, we can use the arbitrary letters *p, q,* and *r.* With this basis, I will attempt to show certain configurations of large-scope English *if-then* statements (first, using a propositional logic overlay) and determine whether those configurations maintain their coherence when we look at them through their English counterparts. As you will see, propositional logic can successfully represent certain large-scope English *if-then* statements, but when logical operators are manipulated, its representation of these statements can easily become unnatural.

First, let's look at two different formulas: $p \rightarrow (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$.

From the truth table below, it can be seen that the two formulas have the same overall truth values when compared side-by-side using the same values for *p, q,* and *r* in each (relevant columns are in italics):

p	*q*	*r*	$p \rightarrow (q \rightarrow r)$	$(p \wedge q) \rightarrow r$
0	*0*	*0*	0 *1* 0 1 0	0 0 0 *1* 0
0	*0*	*1*	0 *1* 0 1 1	0 0 0 *1* 1
0	*1*	*0*	0 *1* 1 0 0	0 0 1 *1* 0
0	*1*	*1*	0 *1* 1 1 1	0 0 1 *1* 1
1	*0*	*0*	1 *1* 0 1 0	1 0 0 *1* 0
1	*0*	*1*	1 *1* 0 1 1	1 0 0 *1* 1
1	*1*	*0*	1 *0* 1 0 0	1 1 1 *0* 0
1	*1*	*1*	1 *1* 1 1 1	1 1 1 *1* 1

I will provide some examples to test whether this rule holds true for English statements. For the purposes of these examples,

$p \rightarrow (q \rightarrow r)$ will be 'Formula A' and $(p \wedge q) \rightarrow r$ will be 'Formula B.'

1 If it rained, then if the sun hasn't come out yet, then the streets are wet.

We can break up the sentence in Example (1) and assign its parts to our arbitrary letters, *p, q,* and *r.* In this case, *p* would be "it rained," *q* would be "the sun hasn't come out yet," and *r* would be "the streets are wet." However, unlike more common English *if-then* statements, this example uses an *if-then* statement as the consequent of a larger *if-then* statement (hence the structure of Formula A, where *p* is the antecedent).

Now, according to the above truth table, Formula B should also be able to successfully represent Example (1) in English, just as Formula A did. A small change in the wording is necessary, but any native speaker of English will attest to there being no change in meaning (as you will see in Example (2) below).

2 If it rained and the sun hasn't come out yet, then the streets are wet.

From Example (2), we can see that Formula A and Formula B are in fact equivalent, and no sentence meaning is lost when they are used to represent large-scope English *if-then* statements.

Now, we can add a third formula to the mix: *(p → q) → r* will be 'Formula C.'

Formula C's truth values with respect to the same values used for *p, q,* and *r* in Formulas A and B are below.

p	*q*	*r*	*(p → q) → r*
0	*0*	*0*	0 1 0 *0* 0
0	*0*	*1*	0 1 0 *1* 1
0	*1*	*0*	0 1 1 *0* 0
0	*1*	*1*	0 1 1 *1* 1
1	*0*	*0*	1 0 0 *1* 0
1	*0*	*1*	1 0 0 *1* 1
1	*1*	*0*	1 1 1 *0* 0
1	*1*	*1*	1 1 1 *1* 1

We can clearly see that the truth values for Formula C differ from those of Formulas A and B. This is supported using an example with our previously defined *p, q,* and *r* from Examples (1) and (2):

3 If it is the case that if it rained, then the sun didn't come out, then the streets are wet.

While Example (3) is an abstract sentence of English and certainly one uncommonly said, it can help to first view it from the perspective of propositional logic: we will treat *(p → q)* as the antecedent of the statement. So, first we have to narrow our focus to just the antecedent. Depending on the truth value of the antecedent, the truth value of the larger *if-then* statement (the whole sentence) will vary.

To determine the truth value of the whole sentence, it makes sense to first evaluate *(p → q)* and then proceed as normal. Applying this to Example (3), "if it rained, then the sun didn't come out" is the antecedent of the sentence, and "the streets are wet" is the consequent. However, it can quickly be realized that it's impossible for a real-world English speaker to determine the truth value of the antecedent (since it can't logically be inferred that the sun didn't come out based simply on the fact that it rained), thus making the whole sentence quite unnatural both to utter and to evaluate logically.

Propositional logic is clearly capable of representing a large-scope English *if-then* statement as shown in Examples (1) and (2). Yet, given the propositions assigned to *p, q,* and *r* in my example, propositional logic's interpretation of Example (1) is distorted and unnatural when we manipulate the symbols as seen in Formula C. This is because in English, it's uncommon to use an *if-then* statement as the antecedent of a larger *if-then* statement (something that can surely be done without issues in propositional logic alone).

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