Variables, Predicates and the Scope of Negation

Before the introduction of variables in our predicate-based sentence meaning evaluations, we were limited to sentences that only used proper names of individuals as arguments of verb predicates. Our approach was not narrow enough to focus on determiners such as *'a'* in *'a librarian.'*

1 John slept.

For instance, the sentence in Example 1 could be represented in predicate form as *slept(John).*

Now, after the introduction of variables into the mix, we're able to represent more complex sentences. Since we can now differentiate between, for example, 'John' and some arbitrary man (*'a man'*), we can do the same for objects, which widens the selection of available sentences we can evaluate using our system.

2 A librarian put a book on a shelf.

Assuming the following context:

- There are three people: Mary, John, Peter.
- Mary and John are librarians.
- There are two shelves: Shelf #1 and Shelf #2.
- There are three books: *Tarzan, Moby Dick, Hamlet.*
- Mary puts *Tarzan* on Shelf #2.

Then Example 2 indeed holds, which can be easily illustrated. Mary is a librarian, *Tarzan* is one book of the three present (*a* book), and that book was put on one shelf of the two present (*a* shelf). These objects in their predicate forms, respectively, are: *librarian(Mary), book(Tarzan), shelf(Shelf).* In this case, we are using "Shelf" as a variable which is acting as the argument of the predicate of the same name, *"shelf()."* Specifically, these three predicates are used to describe the roles of the three variables we've identified as *Mary, Tarzan,* and *Shelf.*

However, the meaning of the sentence in Example 2 cannot be described with these three predicates alone; we need to use them synergistically with our original variable-less approach to predicates, as in Example 3 below:

3 $put(x, y, z) \wedge librarian(x) \wedge book(y) \wedge shelf(z)$

Example 3 is streamlined for the sake of consistency in the variables (x, y, and z) but to relate the example more closely to our context, we can simply switch some variables:

4 $put(Mary, Tarzan, z) \wedge librarian(Mary) \wedge book(Tarzan) \wedge shelf(z)$

Now, in Example 4, 'Shelf' is being represented by the variable *z.* The first part of the formula, *put(Mary, Tarzan, z)*, is familiar since it displays our original variable-less approach to predicates (i.e. "*Mary* put

Tarzan on *z*.”). What’s important is that now we are able to further define the three arguments in the predicate, *put()*, by conjoining predicates via the propositional logic operator, ∧.

To take things further, we can look at the negation of our sentence (“A librarian did *not* put a book on a shelf.”) to see how it can be represented using variables and predicates (keeping with our original context).

5 ¬ *(put(Mary, Tarzan, z)* ∧ *librarian(Mary)* ∧ *book(Tarzan)* ∧ *shelf(z))*

6 ¬ *put(Mary, Tarzan, z)* ∧ *librarian(Mary)* ∧ *book(Tarzan)* ∧ *shelf(z)*

Examples 5 and 6 contain two different possibilities for the negation of our sentence that we can address individually. In Example 5, the negation operator, ¬ is positioned on the outside of a set of parentheses containing our original predicate formula. Using this option negates the truth value of the entire sentence evaluation. In other words, since our original formula evaluated to *true* in our context, simple negating the entire formula would not equate to the addition of “did not” in our sentence “A librarian *did not* put a book on a shelf.” The predicates *librarian(Mary), book(Tarzan),* and *shelf(z)* do not need to be negated at all; they remain true whether a librarian put a book on a shelf or not, since they are all unconditionally present in our context. Specifically, the negation operator has scope over the entire formula, not only the predicate *put().* For these reasons, Example 5 is not an acceptable method of negating our sentence.

Rather, Example 6 is the correct option for the negation of our sentence. Since the negation operator is positioned immediately before the predicate *put()*, it only has scope over *put().* The predicates *librarian(Mary), book(Tarzan),* and *shelf(z)* retain their truth values (as they should, since our context doesn’t change). In English, this translates to “A librarian did not put a book on a shelf.”

Originally, we were limited in our approach to evaluating sentence meaning with predicates since we couldn’t specify variables. This meant that we couldn’t be too specific in terms of the context for the sentences. Now, with the addition of variables, as in *librarian(Mary)*, we’re able to narrow our focus to contextual determiners like ‘*a’* and ‘*the.’*

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