Scope of Universal Quantifiers

Sentences of English containing quantifiers such as *every, some,* and *all,* can commonly be ambiguous in meaning for a native speaker of English. What this means for our analyzation is simply that they can be written as a few different quantified predicate formulas. However, we'll see that the use of two universal quantifiers in a given sentence won't cause the ambiguity that only one would cause.

1 Some student handed in every homework.

In Example 1, which is a typical example of an ambiguous sentence, there are two quantifiers (one of which is a universal quantifier: *every*). Its ambiguity can be illustrated by two separate readings, each of which are perfectly fine interpretations:

2a For <u>every</u> homework, there is <u>some</u> student who handed it in.

2b There is <u>some</u> student who handed in <u>every</u> homework.

When the interpretations in Example 2 are written in their predicate forms, we simply need to switch the quantifier order at the start of the formula (shown in Example 3 below). As it turns out, the first reading follows the *surface scope* interpretation, while the second follows the *inverse scope* interpretation. In the *surface scope,* the order of the quantifiers matches the order of the variables in the formula, while in the *inverse scope,* they are inverted. This is what creates the ambiguity in the original sentence – the possibility for us to substitute the variables in the formula according to two different quantifier orders.

3a some x, every y [student(x) → (homework(y) ∧ hand_in(x, y))]

3b every y, some x [student(x) → (homework(y) ∧ hand_in(x, y))]

Assume the following context, then refer to the substitution for these two formulas on the final two pages:

- There is a professor (*P*), two homework assignments (*h1* and *h2*), two students named John and Mary (*J* and *M*), and Mary did homework 1 while John did homework 2.

For Example 3a, we will substitute the variables in the context for all of the instances of *x* while *y* remains the same, and for Example 3b we will do the opposite. You will see from the evaluation that the *surface scope* reading in Example 3 is true in this context, while the *inverse scope* reading is false in this context. This has to do with the order of context variable substitutions based on the order of our quantifiers.

The ambiguity we run into with these sentences, however, doesn't occur in sentences of English containing *two* universal quantifiers, as opposed to one.

4 Every student handed in every homework.

Now, in Example 4, we see an instance of two uses of the universal quantifier *every* in the same sentence. Like our original sentence, we can write this sentence as a quantified predicate formula, and then switch its quantifier order:

5a every *x,* every *y* [student(*x*) → (homework(*y*) → hand_in(*x, y*))]

5b every *y,* every *x* [student(*x*) → (homework(*y*) → hand_in(*x, y*))]

Based on our conclusion from our first ambiguous sentence, we should be able to assume that the same ambiguity will be present in the sentence in Example 4. However, the substitution order for this new sentence is irrelevant; there is only one logical interpretation of the sentence, which is that *every* student handed in *every* homework. While the sentence can be written two ways, as in Example 6, each of them entails the other. In other words, if one is true, then the other must be true.

6a For every student, it holds that he/she handed in every homework.

6b For every homework, it holds that every student handed it in.

The use of two universal quantifiers overrules the ambiguity that one would expect to arise based on the scheme for writing a sentence's quantified predicate formula. Instead, only one interpretation is possible, and in our example, it's that *every* student handed in *every* homework.

993 words

__Substitution for 3a:__

$[(\text{student}(h1) \rightarrow (\text{homework}(h1) \land \text{hand\_in}(h1, h1)))$

$\lor (\text{student}(h1) \rightarrow (\text{homework}(h2) \land \text{hand\_in}(h1, h2)))$

$\lor (\text{student}(h1) \rightarrow (\text{homework}(J) \land \text{hand\_in}(h1, J)))$

$\lor (\text{student}(\text{h1}) \rightarrow (\text{homework}(M) \land \text{hand\_in}(h1, M)))$

$\lor (\text{student}(h1) \rightarrow (\text{homework}(P) \land \text{hand\_in}(h1, P)))]$

$\land$

$[(\text{student}(h2) \rightarrow (\text{homework}(h1) \land \text{hand\_in}(h2, h1)))$

$\lor (\text{student}(h2) \rightarrow (\text{homework}(h2) \land \text{hand\_in}(h2, h2)))$

$\lor (\text{student}(h2) \rightarrow (\text{homework}(J) \land \text{hand\_in}(h2, J)))$

$\lor (\text{student}(h2) \rightarrow (\text{homework}(M) \land \text{hand\_in}(h2, M)))$

$\lor (\text{student}(h2) \rightarrow (\text{homework}(P) \land \text{hand\_in}(h2, P)))]$

$\land$

$[(\text{student}(J) \rightarrow (\text{homework}(h1) \land \text{hand\_in}(J, h1)))$

$\lor (\text{student}(J) \rightarrow (\text{homework}(h2) \land \text{hand\_in}(J, h2)))$

$\lor (\text{student}(J) \rightarrow (\text{homework}(J) \land \text{hand\_in}(J, J)))$

$\lor (\text{student}(J) \rightarrow (\text{homework}(M) \land \text{hand\_in}(J, M)))$

$\lor (\text{student}(J) \rightarrow (\text{homework}(P) \land \text{hand\_in}(J, P)))]$

$\land$

$[(\text{student}(M) \rightarrow (\text{homework}(h1) \land \text{hand\_in}(M, h1)))$

$\lor (\text{student}(M) \rightarrow (\text{homework}(h2) \land \text{hand\_in}(M, h2)))$

$\lor (\text{student}(M) \rightarrow (\text{homework}(J) \land \text{hand\_in}(M, J)))$

$\lor (\text{student}(M) \rightarrow (\text{homework}(M) \land \text{hand\_in}(M, M)))$

$\lor (\text{student}(M) \rightarrow (\text{homework}(P) \land \text{hand\_in}(M, P)))]$

$\land$

$[(\text{student}(P) \rightarrow (\text{homework}(h1) \land \text{hand\_in}(P, h1)))$

$\lor (\text{student}(P) \rightarrow (\text{homework}(h2) \land \text{hand\_in}(P, h2)))$

$\lor (\text{student}(P) \rightarrow (\text{homework}(J) \land \text{hand\_in}(P, J)))$

$\lor (\text{student}(P) \rightarrow (\text{homework}(M) \land \text{hand\_in}(P, M)))$

$\lor (\text{student}(P) \rightarrow (\text{homework}(P) \land \text{hand\_in}(P, P)))]$

Substitution for 3b:

[(student(*h1*) → (homework(*h1*) ∧ hand_in(*h1, h1*)))

∨ (student(*h2*) → (homework(*h1*) ∧ hand_in(*h2, h1*)))

∨ (student(*J*) → (homework(*h1*) ∧ hand_in(*J, h1*)))

∨ (student(M) → (homework(*h1*) ∧ hand_in(*M, h1*)))

∨ (student(*P*) → (homework(*h1*) ∧ hand_in(*P, h1*)))]

∧

[(student(*h1*) → (homework(*h2*) ∧ hand_in(*h1, h2*)))

∨ (student(*h2*) → (homework(*h2*) ∧ hand_in(*h2, h2*)))

∨ (student(*J*) → (homework(*h2*) ∧ hand_in(*J, h2*)))

∨ (student(*M*) → (homework(*h2*) ∧ hand_in(*M, h2*)))

∨ (student(*P*) → (homework(*h2*) ∧ hand_in(*P, h2*)))]

∧

[(student(*h1*) → (homework(*J*) ∧ hand_in(*h1, J*)))

∨ (student(*h2*) → (homework(*J*) ∧ hand_in(*h2, J*)))

∨ (student(*J*) → (homework(*J*) ∧ hand_in(*J, J*)))

∨ (student(*M*) → (homework(*J*) ∧ hand_in(*M, J*)))

∨ (student(*P*) → (homework(*J*) ∧ hand_in(*P, J*)))]

∧

[(student(*h1*) → (homework(*M*) ∧ hand_in(*h1, M*)))

∨ (student(*h2*) → (homework(*M*) ∧ hand_in(*h2, M*)))

∨ (student(*J*) → (homework(*M*) ∧ hand_in(*J, M*)))

∨ (student(*M*) → (homework(*M*) ∧ hand_in(*M, M*)))

∨ (student(*P*) → (homework(*M*) ∧ hand_in(*P, M*)))]

∧

[(student(*h1*) → (homework(*P*) ∧ hand_in(*h1, P*)))

∨ (student(*h2*) → (homework(*P*) ∧ hand_in(*h2, P*)))

∨ (student(*J*) → (homework(*P*) ∧ hand_in(*J, P*)))

∨ (student(*M*) → (homework(*P*) ∧ hand_in(*M, P*)))

∨ (student(*P*) → (homework(*P*) ∧ hand_in(*P, P*)))]